SEMESTRAL EXAMINATION
ANALYSIS IV, B. MATH II YEAR
II SEMESTER, 2012-2013

Max. you can score: 100 Time limit: 3 hrs.

The six questions below carry a total of 110 marks. The maximum you can score is 100. Answer as many questions as you can.

1. Let $f : (a, b) \to \mathbb{R}$ be a continuous function such that $f(\frac{x+y}{2}) \leq \frac{f(x)+f(y)}{2}$ $\forall x, y \in (a, b)$. Show that f is convex. [20]

2. Let $\{f_n\}$ be a sequence of maps from $\mathbb{R}$ to $\mathbb{R}$ which is equicontinuous and uniformly bounded. Prove that there is a subsequence $\{f_{n_j}\}$ which converges pointwise to a continuous function on $\mathbb{R}$. [20]

3. Let $\{f_n\}$ be a sequence of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ which is pointwise bounded. Prove that for any $a < b$ the interval $[a, b]$ contains an open interval on which the sequence is uniformly bounded. [15]

4. Let $f(x) = |\sin x|, x \in \mathbb{R}$. Write down the Fourier series of f and prove that it converges to f at every point. [20]

5. Let $f(c) = \sum_{n=0}^{\infty} a_n c^n$ for all $c \in \mathbb{C}$ with $|c| \leq 1$ where $\{a_n\}$ is a sequence of complex numbers such that $\{n^2 a_n\}$ is bounded. Show that $f(c) = 0$ whenever $|c| = 1$ implies $f(c) = 0$ for all $c \in \mathbb{C}$ with $|c| \leq 1$. [15]

6. Prove that $\cos x = \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{n \sin(2nx)}{4n^2-1}$ if $0 < x < \pi$. [20]